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Demonstrating cosmic ray induced electromagnetic cascades in the A-level laboratory

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This article indicates how the study of sea-level cosmic ray phenomena can have a role in A-level physics. It describes a simple but far reaching particle physics experiment that can be carried out in the A-level physics laboratory. A simple model of electron–positron–photon cascades, suitable for use at A-level, is described.

Particle physics is destined to become a standard topic in A-level physics. Excellent curriculum support packages have been produced by the Institute of Physics [1] and the Physics Department at the University of Lancaster [2], and there is a growing range of high quality textbooks aimed at the A-level audience. But the nature of the topic makes it difficult to develop direct laboratory experience for the students.

The current understanding of the standard model of particle physics has grown out of accelerator-based investigations carried out over the last 40 years in multinational collaborations and at great expense. It is understandable that A-level presentations should be based upon that work and the absence of appropriate sixth-form laboratory experience is taken for granted. ‘Hands-on’ work at A-level currently involves using computer simulations of particle tracks or interpreting data from accelerator experiments.

The foundations of modern particle physics are to be found in the cosmic ray studies of the 1930s and 1940s. The investigations of that era were carried out in ordinary laboratories with relatively straightforward apparatus by today’s standards. In the 1950s, particle physicists turned away from cosmic ray studies towards accelerator-based work because it freed them from the serendipitous nature of the investigations; accelerators allowed researchers to create particles and study their interactions in a controlled way. But serendipity is not an obstacle to studies in A-level laboratories.

Cosmic rays provide a relatively constant flux of high energy particles at sea level with a density of about one particle per square centimetre per minute. This high energy flux is free for our use and our explorations are limited only by the availability of low-cost detection and signal processing apparatus. With appropriate interfacing, today’s personal computers can be turned into recorders, controllers, counters, timers, multi-channel analysers etc. Advances in particle detection technology should be able to deliver good quality probes suitable for use in the A-level laboratory in the near future. If we can combine the three strands—the available cosmic ray flux, the powerful computers sitting in most physics teaching laboratories and the prospects of modern detection apparatus—then we might be able to see A-level students carrying out a range of particle physics investigations as a matter of routine.

On the other hand, some exotic cosmic ray effects can be investigated using standard apparatus found in most A-level laboratories. This article describes how three Geiger–Müller (GM) tubes can be combined with a simple coincidence circuit to demonstrate photon–electron–positron cascades and how the interpretation of the results draws upon mathematics and theory of the type found in current A-level courses.

Cosmic rays

The Earth is being continuously bombarded by high energy particles from deep space. The particles are mainly high energy protons together with a small component of helium nuclei, heavier nuclei and electrons. The energy range is enormous with some particles having energies of the order of 10^{20} eV.

These primary cosmic rays hit the upper atmosphere at a rate of about 1000 per square metre per second. They collide with atoms in the atmosphere and produce large showers of newly created secondary particles, the progeny of which can be detected at ground level. The initial interactions in the upper atmosphere produce large numbers of charged and neutral pions. The charged pions decay into muons and muon-neutrinos whereas the uncharged pions decay into pairs of high energy photons, which become the starting points of large cascades of electrons, positrons and gamma rays. The resulting flux of particles at ground level consists mainly of these muons and electrons/positrons in the ratio of roughly 75% to 25%.

The muons lose their energy gradually by ionization of the material through which they pass. As they start with high energies they have the capacity to ionize many atoms before their energy is exhausted. Also, as they travel at nearly the speed of light, they tend not to ionize very efficiently and hence can travel through substantial lengths of matter, some metres of lead, before being stopped.

The process is a little different for the electrons. Theory shows that ionization of the medium is not the predominant energy loss mechanism for relativistic electrons; high energy electrons lose energy more efficiently through the emission of electromagnetic radiation as they are decelerated in the presence of matter. If ionization had been the main mechanism for slowing the electrons down then they might have had penetrative properties comparable to those of the muons, but the radiative energy loss mechanism ensures that about 15 cm of lead is sufficient to stop them.

The different energy loss mechanisms allow the sea-level particle flux to be split into two main components. The penetrating muon part of the ground level flux is referred to as the ‘hard’ component and the easily absorbed electron part is called the ‘soft’ component.

Electromagnetic cascades

The decay of a neutral pion into a pair of high energy gamma rays is the starting point of an avalanche of electrons, positrons and further gamma rays. This avalanche is known as an electromagnetic cascade. The process starts when one of the gamma rays passes close to the nucleus of an atom. Even though the gamma ray carries no electric charge its electromagnetic nature allows it to interact with the strong electric field of the nucleus to cause the materialization of an electron–positron pair. The energy required for pair creation is about 1 MeV; the gamma rays can have a thousand times that energy and hence the electron–positron pair produced can move on, sharing nearly all the energy of the initiating gamma ray. If these fast moving electrons and positrons go on to pass close to other nuclei then they will suffer accelerations due to the positive charge of the protons. An accelerated charged particle will emit electromagnetic radiation. The intense accelerations can produce further gamma rays capable of producing more electron–positron pairs. The cycle of pair production and gamma ray generation continues, with the original gamma ray energy eventually manifesting itself as many particles. The process is shown schematically in figure 1.

The cascade generation ceases when the shares of energy get sufficiently small such that the electrons are no longer capable of radiating efficiently; the relatively slow moving electrons are then brought to rest by ordinary ionization processes. The electron energy at which the main energy loss mechanism changes from radiation losses to ionization losses is known as the critical energy E_c. The critical energy for electrons in lead is about 7.6 MeV.

Whilst the electromagnetic cascade can grow over a large distance in air, it is confined to much smaller regions in solids, where the number of atoms per centimetre of path length is greater. If the material is made up of atoms with a high

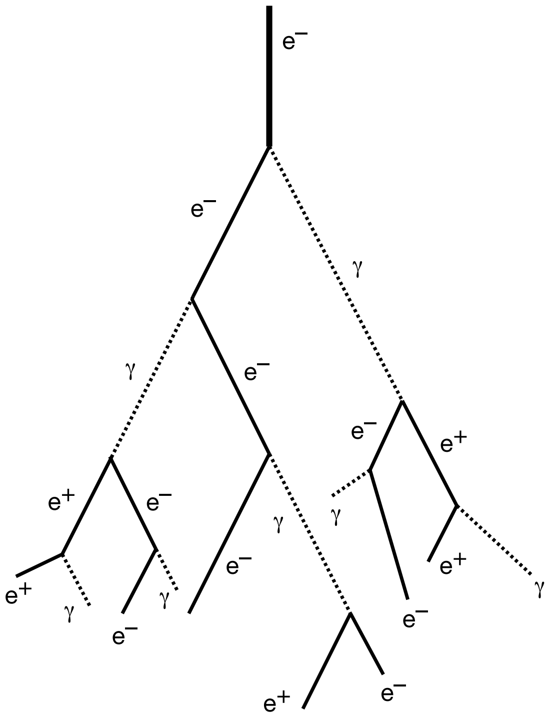

Figure 1. A schematic representation of an electron-initiated electromagnetic cascade.

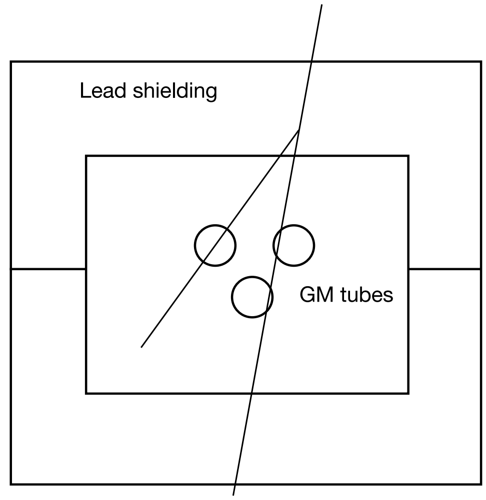

Figure 2. Rossi's apparatus for investigating cosmic ray induced showers in lead.

atomic number then the greater nuclear charges can produce greater accelerations and so the cascade process can develop more readily than it would in a material with a lower atomic number.

Electromagnetic cascades in lead

In his 1964 popular account of the historical development of the understanding of the nature of the cosmic radiation [3], Bruno Rossi describes the crucial experiments that led to today's picture. He recounts how, at the age of 24 in 1930, he built his own Geiger–Müller counters and developed vacuum tube coincidence electronics to enable him to use groups of GM tubes in an investigation into the cosmic ray penetration of dense materials. He found that a substantial proportion of the cosmic radiation detected at sea level could penetrate over 1 m of lead.

About the same time, investigators using cloud chambers reported observations of multiple tracks associated with the apparent generation of secondary particles close to the walls of the chamber. Rossi pursued this by using a coincidence arrangement of three GM tubes in a triangular array within a lead enclosure in order to record the generation of groups of particles (figure 2). A triangular arrangement was used to cut out the possibility of a single particle discharging all the counters at the same time. He was surprised to record a high rate of coincidences, as many as 35 per hour in some cases. When the upper part of the lead shielding was removed the coincidence rate fell to about two coincidences per hour. Even this lower coincidence rate was surprising since counter statistics associated with random coincidences predicted a much lower value (see appendix A). The background coincidence rate can now be understood in terms of showers of particles produced in the air above the counters.

The investigations were developed to see how the coincidence rate varied with thickness of lead placed above the counters. It was found that the rate increased steadily with thickness of lead, reaching a maximum between 1 and 2 cm then falling steadily to about half of the peak value as the lead thickness increased to about 5 cm (figure 3). The rapid fall-off of the coincidence rate beyond the peak of the shower curve made it clear that the particles responsible for the production of the groups of particles were not the same as those capable of penetrating 1 m of lead with ease.

The explanation of the processes involved in electromagnetic cascades developed in the years

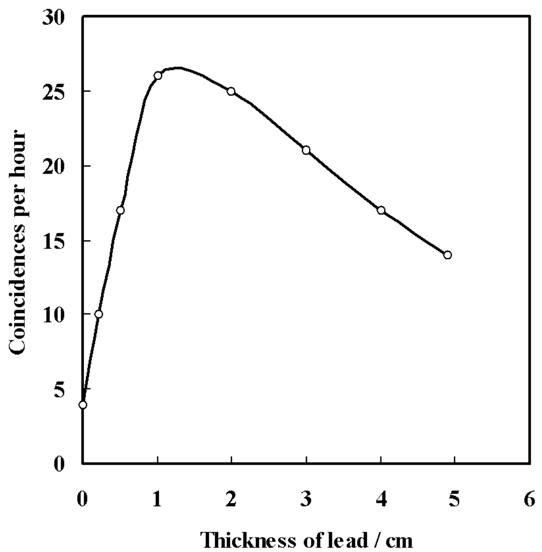

Figure 3. Shower curve obtained by Rossi using a triangular arrangement of Geiger–Müller tubes.

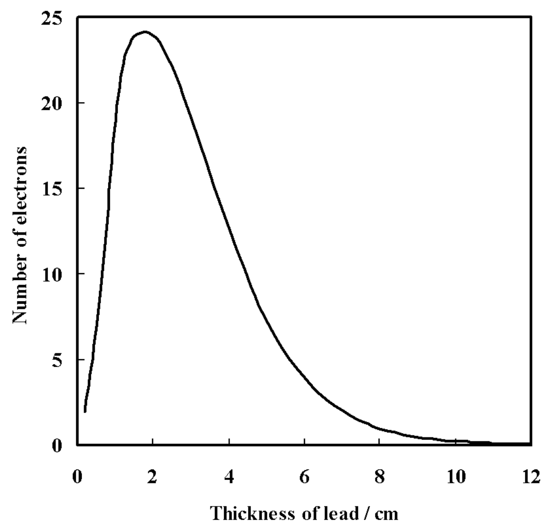

Figure 4. An example of a theoretical prediction of the variation of the number of shower particles beneath a lead absorber as a function of thickness of lead (from Rossi [3]).

that followed. Dirac had already predicted the existence of positrons and Carl Anderson produced cloud chamber evidence of positive electrons. In 1934 Hans Bethe and Walter Heitler considered in detail what happened when a charged particle passed through the strong electric field of an atomic nucleus. They found that, at high energies, the particle is likely to release a large fraction of its energy as a photon. The picture of the deceleration-induced emission of high energy gamma rays (called bremsstrahlung, or braking radiation) and the subsequent materialization of an electron–positron pair (pair production) began to develop. The theory enabled physicists to calculate the typical numbers of electrons and positrons likely to emerge below a given thickness of lead when a relativistic charged particle or a high energy gamma ray entered it from above. The graphs of the number of shower electrons against lead thickness had characteristics similar to those of the number of coincidences recorded against absorber thickness (figure 4). One can expect the two curves to have similar shapes since the probability of recording a coincidence with the three GM tubes is directly related to the number of electrons appearing below the plates.

The analysis of many sets of experimental data showed that the cascade theory adequately explained the features of the showers.

Reproducing the cascade experiment in the A-level laboratory

Rossi's cascade coincidence experiment lends itself well to reproduction in the A-level laboratory. Suitable lead absorbers can be made from lead flashing used in the building industry. It works satisfactorily with standard GM counters connected in a coincidence arrangement through an ordinary AND gate. The coincidence rates are smaller than those reported by Rossi but the shower curve develops well if long timing intervals are used.

Our exploratory investigations gave satisfactory results at the first try. We obtained 20 cm $\times$ 20 cm sheets of lead flashing, each with a thickness of about 1.8 mm, from the Construction department of the college. Three MX168 GM tubes were arranged at first in a triangular array and then later in a crossed arrangement (figure 5). Photographs of the apparatus are shown in figure 6. The configurations were such that a single particle was unlikely to trigger all three GM tubes at the same time. The size of the GM tube holders dictated the spacing of the tubes in the triangular arrangement; the crossed arrangement allowed the tubes to be placed more closely together and produced a higher coincidence rate. Both arrangements gave a comparable coincidence rate in the absence of

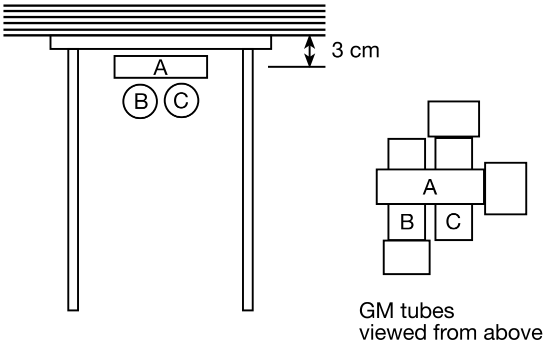

Figure 5. Experimental arrangement for reproduction of the shower curve in the A-level laboratory.

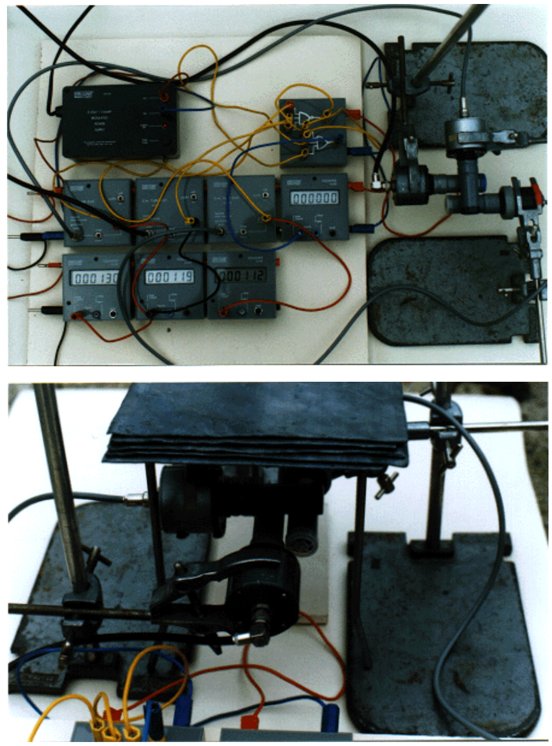

Figure 6. (*a*) The GM tube arrangement and associated counters. The laboratory stand and the lead plates have been removed from their position above the GM tubes. (*b*) Relative positions of the laboratory tripod and lead plates. The GM tubes have been lowered from their position 3 cm below the plates so that they can be seen in the photograph.

shower-producing absorbers.

The lead absorbers were supported by a standard laboratory bunsen burner tripod. The GM tubes were driven by Unilab GM tube EHT units, each having a pulse output of height 4 V and a measured width of about 30 μs. The three pulse outputs were connected to a three-input AND gate formed using two NAND gates of a 7410 chip. The output of the AND gate then drove a Unilab digital display.

Calculation showed that the expected number of 'accidentals', spurious coincidences due to three unrelated GM tube pulses occurring at the same time, would be of the order of 5×10^{-7} per hour, i.e. the accidental rate would be negligible (see appendix A).

Because the maximum count rate obtained by Rossi occurred between 1 cm and 2 cm of lead we placed ten sheets of the flashing on the tripod and obtained a coincidence rate of about four per hour. We then removed the ten sheets and ran the arrangement for a full day with no absorbers. We got a count of eight coincidences in 24 hours without the lead. We accepted that our apparatus was simply not as sensitive as Rossi's but decided to persevere. We initially decided to count for 24 hours for each reading and increase the lead thickness at a rate of two sheets per day. It soon became clear that whilst the overall count rate was low, our rough and ready experiment produced a shower curve with the same characteristics as those shown in Rossi's account. We found that placing the GM tubes about 3 cm beneath the lead produced a higher count rate and allowed us to count for 12 hour intervals. A typical graph is shown in figure 7.

The peaks of the graphs obtained occurred at about ten sheets (18 mm) in the same area of the graph as found by Rossi. The simplified theory of the process, described in appendix B, suggests that the peak of the graph should occur at a thickness t_{max} given by the equation

$$t_{\mathrm{max}} = \frac{\ln(E_0/E_{\mathrm{c}})}{\ln 2}$$

where t is measured in radiation lengths L_{R}, E_0 is the energy of the incoming particle and E_{c} is critical energy for the material, below which the electrons tend to lose energy by simple ionization rather than by bremsstrahlung processes. The equation rearranges to

$$2^t E_{\mathrm{c}} = E_0.$$

Using the accepted values of $L_{\mathrm{R}} = 0.56$ cm and $E_{\mathrm{c}} = 7.6$ MeV for electrons in lead, a graph peak

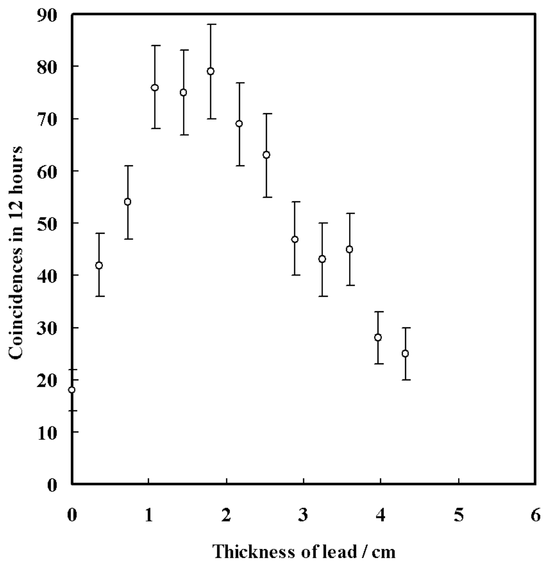

Figure 7. Example of an experimental shower curve obtained in the A-level laboratory.

at 1.8 cm corresponds to 3.21 radiation lengths. The equation above yields a value for the average energy of the incident electrons of the order of 70 MeV. This is an impressive value when one considers that the standard radioactive sources used at A-level have energies of no more than a few MeV.

Rossi reported a peak coincidence rate about four times as much as ours. We assume that he might have had a greater collection area and/or GM tubes that were more sensitive to the radiation. Our MX168 tubes have a sensitive length of about 4 cm and a diameter of about 2 cm, giving an effective collection area of about 8 cm^2 per tube. In his 1952 book, *High Energy Particles* [4], Rossi describes typical GM tubes used in early cosmic ray work with a length of the order 30 cm and diameter 2 cm, giving an effective collection area about seven times greater than ours; it is likely that the difference in count rate can be accounted for by the differences in GM tube size and the sensitivities of the detectors.

Conclusion

Rossi's electromagnetic cascade experiment can be reproduced easily in the standard A-level laboratory and gives striking evidence of the existence of the high energy radiation all around us. The experiment can complement discussion of electron–positron pair production. It gives a context for discussing charged pion decay into muons and neutrinos, and for neutral pion decay into two gamma rays. The discussion can be taken further to draw in the need for a nucleus interaction in order to conserve energy and momentum when the electron–positron pair materialize out of the gamma ray energy.

The work could even be stretched to the consideration of the energy loss mechanisms in the passage of charged particles through matter. Current A-level students touch on this topic when they calculate the number of ions per cm track length when relatively slow moving alpha particles from radioactive sources pass through air. A qualitative account at A-level could cover the decrease in ionization cross-section together with the increasing probability of radiative energy losses as the particle velocity increases.

Similarly, students could learn how low energy photons lose energy by Compton scattering whereas high energy photons lose energy by pair production.

The simple model of electromagnetic cascades outlined in appendix B is no more mathematically demanding than the standard A-level topic of radioactive decay. Combined with a general discussion of cosmic rays and the laboratory generation of the Rossi shower curve, it could form a useful component of a syllabus geared towards building upon the discoveries of this century as a preparation for the next.

Appendix A. Calculating the rate of 'accidentals' in coincidence arrangements

'Accidental' coincidences occur when unrelated pulses from the detectors coincide by chance. Simple probability arguments can be used to produce an estimate of the expected coincidence rate.

Two counters

We assume that each counter has output pulses of width $\mathrm{d}t$. The counters are labelled A and B. Let us assume that a pulse arrives at counter A. If counter B produces a pulse within a time

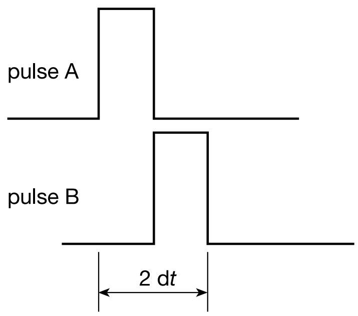

Figure A1. Coincidence interval for two pulses of width d*t*.

interval $2\,\mathrm{d}t$ following the start of pulse A then a coincidence will be recorded; the situation is illustrated in figure A1.

If counter A produces n_A pulses in one second then the total time available for B to produce a coincidence will be the fraction $2n_A\,\mathrm{d}t$ of a second. If B produces pulses at a rate n_B per second then over a long time interval we can expect the number of accidental coincidences to be

$$n(\mathrm{A.B}) = 2n_A n_B\,\mathrm{d}t.$$

Extending the argument to three counters A, B, C

We assume that counter A produces a pulse first. The probability that the pulse from B will coincide with that from A will be given by

$$\frac{n(\mathrm{A.B})}{n_A} = 2n_B\,\mathrm{d}t.$$

By an argument similar to that given above, the probability that a pulse from a third counter, C, will occur within a time interval $2\,\mathrm{d}t$ of the start of pulse A will be given by the expression

$$\frac{n(\mathrm{A.C})}{n_A} = 2n_C\,\mathrm{d}t.$$

The probability that both B and C will occur within $2\,\mathrm{d}t$ of A is then the product of the two probabilities, i.e.

$$P(\mathrm{A.B.C}) = \frac{n(\mathrm{A.B.C})}{n_A} = 2n_B\,\mathrm{d}t \times 2n_C\,\mathrm{d}t.$$

The expected number of accidentals with three counters acting in coincidence will then be

$$n(\mathrm{A.B.C}) = 4n_A n_B n_C\,\mathrm{d}t^2.$$

However, this turns out to be a bit of an oversimplification: it has ignored situations where two of the counters might be triggered together by a single particle passing through them and then produce a chance coincidence with the third. This kind of situation turns out to alter the numerical multiplying factor from 4 to a higher value but leaves the overall $\mathrm{d}t^2$ factor intact. A full mathematical treatment [5] that takes into account the Poisson distribution of interpulse times produces the same equation but with the multiplying factor of 4 replaced by 3.

In the experiment described in the article, the pulse width dt had a value of 30 μs. The average count rate from each counter was 20 per minute. Using the equation derived above we get an expected accidental rate of

$$\begin{aligned} n(\mathrm{A.B.C}) &= 4\,(20/60)^3(30\times10^{-6})^2 \text{ per second} \\ &= 1.3\times10^{-10} \text{ per second.} \end{aligned}$$

This becomes an expectation of about 5×10^{-7} accidentals per hour.

Appendix B. Electromagnetic cascade theory

The simple model of the electromagnetic cascade presented here is described in detail in *Particle Physics* by Martin and Shaw [6] and in *High Energy Particles* by Rossi [4].

Radiation length

It can be shown that, for relativistic electrons, the average rate of energy loss per cm of path length due to bremsstrahlung is directly proportional to the particle energy. The relationship is given by the equation

$$\frac{\mathrm{d}E}{\mathrm{d}x} = -\frac{1}{L_R}E \qquad \text{(B1)}$$

where L_R is called the radiation length of the material.

The radiation length for electrons of mass m passing through a material of density n_a atoms per cm^3 and atomic number Z is given by the equation

$$\frac{1}{L_R} = 4\left(\frac{h}{2\pi mc}\right)^2 Z(Z+1)\alpha^3 n_a \ln\left(\frac{183}{Z^{1/3}}\right) \qquad \text{(B2)}$$

where α (= 1/137) is the fine structure constant.

From equation (B1) we can see that a particle entering an absorber with an initial energy E_0 will have an energy given by

$$E_x = E_0 \exp(-x/L_{\mathrm{R}})$$

after passing through x cm of absorber.

The radiation length can be interpreted as the average thickness of material that reduces the mean energy of the charged particle by a factor e. The value of L_{R} for electrons in lead is 0.56 cm, for iron it is 1.8 cm and for aluminium it is 8.9 cm.

It can be seen from equation (B2) that L_{R} is proportional to mass^2. Because the mass of the muon is some 200 times that of the electron, it can be expected to have a radiation length much greater than that of the electron. This explains why muon energy loss by bremsstrahlung is negligible compared with that of electrons, and hence the muons are capable of traversing much greater lengths of absorbing material before being brought to rest.

High energy photon absorption through pair production also turns out to have an exponential character. The rate of attenuation of a collimated beam of I photons per second is described by the equations

$$\frac{\mathrm{d}I}{\mathrm{d}x} = -\frac{1}{\lambda} I \qquad \text{and} \qquad I_x = I_0 \exp(-x/\lambda)$$

where λ is the mean free path for the photons in the absorber.

The mean free path for photon absorption through pair production is related to the radiation length for electron bremsstrahlung by the equation

$$\lambda = \tfrac{9}{7} L_{\mathrm{R}}.$$

It can be seen that the two characteristic lengths are comparable in magnitude.

The general features of the electromagnetic cascade can be accounted for by a simplified model of the successive generation of photons and electron–positron pairs.

The simplified cascade model

The model is based upon the following restrictions:

(i) the incoming charged particles have a starting energy E_0 that is much greater than the critical energy, E_{c}, below which ionization losses predominate over pair production, i.e. $E_0 \gg E_{\mathrm{c}}$;

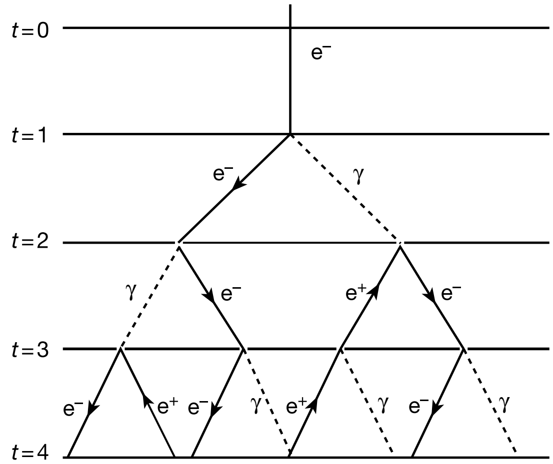

Figure B1. The simplified electromagnetic cascade model. Values of t represent successive radiation lengths.

(ii) we assume that each electron with $E_0 > E_{\mathrm{c}}$ travels one radiation length and then gives up half of its energy to a bremsstrahlung photon;

(iii) we assume that each photon produced with energy $E_\gamma > E_{\mathrm{c}}$ travels one radiation length and then creates an electron–positron pair with each particle carrying away half the energy of the original photon;

(iv) electrons with $E < E_{\mathrm{c}}$ cease to radiate and then lose the rest of their energy by collisions.

The process is illustrated in figure B1.

This simple branching model suggests that after t radiation lengths the shower will contain 2^t particles. There will be roughly equal numbers of electrons, positrons and photons, each with an average energy given by

$$E(t) = E_0/2^t.$$

The cascading process will stop abruptly when $E(t) = E_{\mathrm{c}}$.

The thickness of absorber at which the cascade ceases, t_{max}, can be written in terms of the initial and critical energies, i.e.

$$E_{\mathrm{c}} = E_0/2^t$$

hence

$$2^t = E_0/E_{\mathrm{c}}$$

and

$$t_{\mathrm{max}} = \frac{\ln(E_0/E_{\mathrm{c}})}{\ln 2}.$$

The model suggests that the maximum shower depth varies as the logarithm of the primary energy, a feature that emerges from more sophisticated models of the process and is observed experimentally. It also predicts that the shower curve should rise rapidly to a peak value and then fall to zero. The broad peak of the experimental curve can be interpreted in terms of a spread of energies of the incoming particles. Experiment also shows that the curve does not eventually drop to zero but instead has a long tail. The long tail can be interpreted as being due to muon interactions producing knock-on electrons capable of making a contribution to the cascade process.

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